



education

Department of
Education
FREE STATE PROVINCE

PREPARATORY EXAMINATION

GRADE 12

MATHEMATICS P1

SEPTEMBER 2020

TIME: 3 HOURS

MARKS: 150

This paper consists of 7 pages and 1 information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. The question paper consists of 11 questions.
2. Answer ALL the questions.
3. Number your answers correctly according to the numbering system used in this question paper.
4. Clearly show ALL calculations, diagrams, graphs, etc. that you have used in determining your answers.
5. Answer only will NOT necessarily be awarded full marks.
6. You may use an approved scientific calculator (non-programmable and non-graphic) unless stated otherwise.
7. If necessary, round off answers to TWO decimal places, unless stated otherwise.
8. Diagrams are NOT necessarily drawn to scale.
9. An information sheet with formulae is included at the end of the question paper.
10. Write neatly and legibly.

QUESTION 1

1.1 Solve for x :

1.1.1 $(x + 5)(x - 3) = -15$ (3)

1.1.2 $3x^2 - 4x - 11 = 0$ (correct to TWO decimal places) (4)

1.1.3 $2x^2 - 3 \geq 5x$ (4)

1.1.4 $\sqrt{2x+1} + 1 + \frac{12}{\sqrt{2x+1} + 3} = 5$ (5)

1.1.5 $\sqrt[3]{x^2} - 4\sqrt[3]{x} - 5 = 0$ (4)

1.2 If $2x^3 - 3x^2 - 17x - 12 = (x + 1)(x - 4)(2x + 3)$, HENCE or otherwise, solve
 $2(y - 2)^3 - 3(y - 2)^2 + 17(2 - y) = 12$ (4)
[24]

QUESTION 2

2.1 Given the quadratic number pattern 1; x ; 1; -2 ; y ; ...; -322

2.1.1 Solve for x and y . (4)

2.1.2 Calculate the number of terms in this pattern. (5)

2.2 Given $S_n = 5n - 3$, determine T_{34} . (3)

2.3 In an arithmetic sequence, the tenth term is 28. The sum of term 5 and term 7 is 32. Calculate the sum of the first 50 terms. (5)
[17]

QUESTION 3

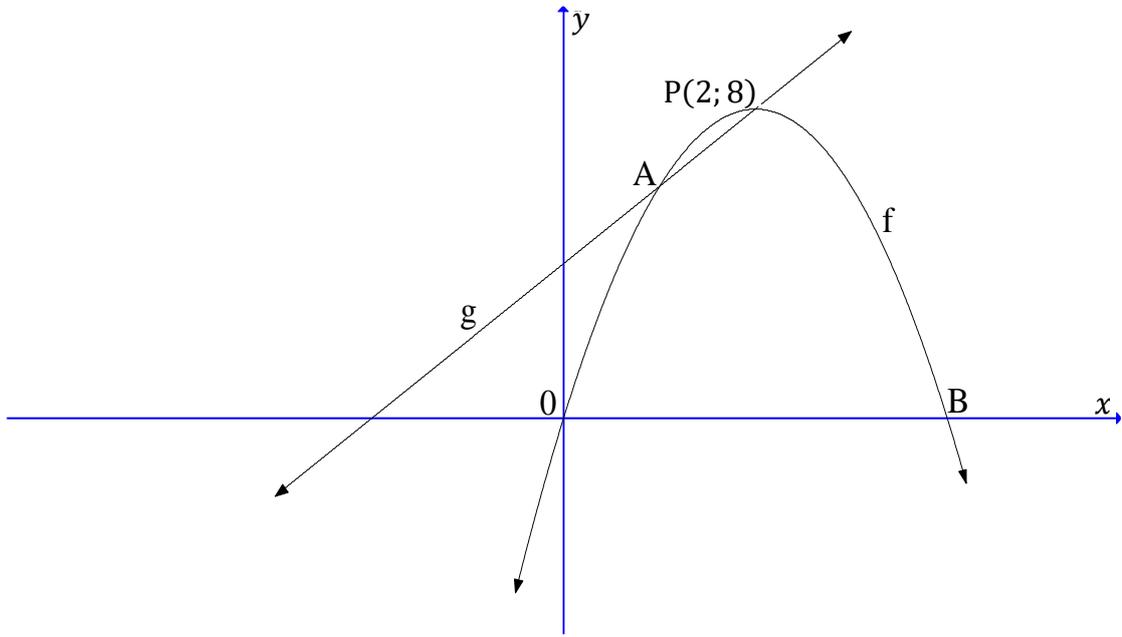
3.1 Prove that in a geometric series with first term a and constant ratio r , the sum of the first n terms is given by $S_n = \frac{a(1 - r^n)}{1 - r}$ (4)

3.2 The first two terms of a geometric series is $\sqrt{3}$ and $\sqrt{3} - 1$. Determine WITHOUT THE USE OF A CALCULATOR, the value of S_∞ . (4)

3.3 In a certain geometric sequence, the sum of terms 3, 4 and 5 equals 28 and the sum of terms 6, 7 and 8 equals 224. Determine the first three terms of this sequence. (5)
[13]

QUESTION 4

- 4.1 The sketch shows the graph of parabola f with turning point $P(2; 8)$ and passing through the origin. The straight line $g(x) = 2x + 4$ passing through A and P is also shown.



- 4.1.1 Prove that the equation of f is $y = -2x^2 + 8x$ (4)
- 4.1.2 Determine the coordinates of A and B. (3)
- 4.1.3 Determine the values of x where $f'(x)g(x) \leq 0$. (2)
- 4.1.4 Determine the range of $f(x+1) - 1$. (2)
- 4.1.5 Calculate the value of x where $g(x)$ will be a tangent to f . (3)
- 4.2 Given the function $h(x) = \frac{2}{x-2} + 1$.
- 4.2.1 Write down the equations of the asymptotes of h . (2)
- 4.2.2 Determine the equation of the decreasing axis of symmetry of $h(x-1)$. (3)
- [19]

QUESTION 5

Given the function $f(x) = 2^{-x}$.

- 5.1 Determine g , the inverse of f , in the form $y = \dots$ (2)
- 5.2 Is g a function? Give a reason for your answer. (2)
- 5.3 Draw sketch graphs of f and g on the same set of axes.
Clearly indicate intercepts with axes and asymptotes on your sketch. (4)
- 5.4 The graph of f is shifted 1 to the LEFT and 2 DOWN to form the graph of h .
Determine the equation of h and write your equation with positive exponents. (2)
- [10]**

QUESTION 6

- 6.1 How long will it take (answer to the nearest year) for the value of an investment to depreciate with a quarter of its original value? Rate of depreciation is 8,2% p.a. on the reducing balance method. (4)
- 6.2 Ina wants to travel overseas in 6 years' time. She will need R58 480 to do that. Calculate her monthly payment into a savings account with an interest rate of 9% p.a. compounded monthly if she makes her first payment immediately and her last payment two months before the end of the 6 years. (5)
- 6.3 Jacob took out a loan of R1 500 000 to buy a house. He will repay the loan with monthly payments over 20 years.
The interest rate is 8% p.a. compounded quarterly.
- 6.3.1 Showing ALL your calculations and formulae, prove that his monthly instalment will be R12 499,96. (5)
- 6.3.2 Calculate the outstanding amount after 12 years. (3)
- [17]**

QUESTION 7

7.1 Given $f(x) = -x^2 - 2$, determine $f'(x)$ using FIRST PRINCIPLES. (5)

7.2 Determine:

7.2.1 $\frac{dy}{dx}$ if $y = \frac{x}{3} + \sqrt[4]{x^3} - 5p^2$ (3)

7.2.2 $D_x \left[(2x^{-1} - \sqrt{5})^2 \right]$ (3)

[11]

QUESTION 8

Given $f(x) = 2x^3 + ax^2 + bx + 3$. The point $(2; -9)$ is a turning point of the function.

8.1 Determine the values of a and b . Show ALL your calculations clearly. (6)

8.2 If it is given that $f(x) = 2x^3 - 5x^2 - 4x + 3$, prove that $(x + 1)$ is a factor of f . (2)

8.3 HENCE draw a sketch graph of f , clearly indicating ALL intercepts and coordinates of turning points. (5)

8.4 For which value(s) of x will the graph be concave down? (2)
[15]

QUESTION 9

A dairy company wishes to market its milk in rectangular carton containers.

The volume of the container must be exactly $1l$, and the length of the base must be twice the breadth of the base. [$1l = 1000cm^3$]

9.1 Show that the height (h) of the container can be written as $\frac{500}{x^2}$, where x is the breadth of the base. (2)

9.2 Determine the dimensions of the container if the surface area must be a minimum. Ignore the thickness of the carton. (4)

[6]

QUESTION 10

10.1 Given $P(A \text{ or } B) = 0,4$; $P(A) = 3P(B)$ and events A and B are independent. Determine $P(B)$. (4)

10.2 Jan goes to the cinema or to a club on a Friday night. He goes to a club 60% of the time and then sleeps late on Saturday morning 70% of the time. If he goes to the cinema, he has a 40% probability of sleeping late on Saturday morning. Determine the probability that Jan sleeps late on a randomly selected Saturday. (3)
[7]

QUESTION 11

11.1 There are 5 chairs on the stage and 3 boys and 2 girls must go on stage.

11.1.1 How many possible ways are there for them to sit on stage? (2)

11.1.2 Calculate the probability that the boys will sit next to each other with the girls at the two ends. (2)

11.2 The digits 3 to 9 are available to set a 4 digit code for you locker at school.

11.2.1 How many different codes are possible if the digits may be repeated and the code must be an even number less than 6 000? (3)

11.2.2 Calculate the probability that the code will start with an odd number and be divisible by 5. The digits may not be repeated. (4)
[11]

TOTAL: 150

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + in)$$

$$A = P(1 - in)$$

$$A = P(1 - i)^n$$

$$A = P(1 + i)^n$$

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1}; r \neq 1$$

$$S_\infty = \frac{a}{1 - r}; -1 < r < 1$$

$$F = \frac{x[(1 + i)^n - 1]}{i}$$

$$P = \frac{x[1 - (1 + i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x + h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan \theta$$

$$(x - a)^2 + (y - b)^2 = r^2$$

In ΔABC : $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$$a^2 = b^2 + c^2 - 2bc \cdot \cos A$$

$$\text{Area } \Delta ABC = \frac{1}{2} ab \cdot \sin C$$

$$\begin{aligned} \sin(\alpha + \beta) &= \sin \alpha \cdot \cos \beta + \cos \alpha \cdot \sin \beta \\ \cos(\alpha + \beta) &= \cos \alpha \cdot \cos \beta - \sin \alpha \cdot \sin \beta \end{aligned}$$

$$\begin{aligned} \sin(\alpha - \beta) &= \sin \alpha \cdot \cos \beta - \cos \alpha \cdot \sin \beta \\ \cos(\alpha - \beta) &= \cos \alpha \cdot \cos \beta + \sin \alpha \cdot \sin \beta \end{aligned}$$

$$\cos 2\alpha = \left\{ \begin{array}{l} \cos^2 \alpha - \sin^2 \alpha \\ 1 - 2\sin^2 \alpha \\ 2\cos^2 \alpha - 1 \end{array} \right\}$$

$$\sin 2\alpha = 2 \sin \alpha \cdot \cos \alpha$$

$$\bar{x} = \frac{\sum x}{n}$$

$$\sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx$$

$$b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$